Collision and Debris Hazard Assessment for a Low-Earth-Orbit Space Constellation

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In an effort to assess issues associated with a low-Earth-orbit constellation deployment, the potential collision of the constellation spacecraft with background trackable and smaller debris objects was examined. The background flux of debris was projected to the year 2002. The collision potential with a Space Station sized object passing through the constellation was also determined. A modified analytical approach was used to estimate the collision probabilities in Earth orbit based on the concepts from the asymptotic theory of extreme-order statistics. The approach is based on a propagation or a Monte Carlo simulation of the orbital motion of a spacecraft and a population of objects in orbit. An empirical set of close approach distances is obtained at random times from which the probability of collision and the rate of encounters at specified ranges is determined. The encounter rate within a specified range was determined and found to compare favorably with that obtained from a close approach determination software package.

Nomenclature

| $A_{\rm SS}$ | = area of Space Station |
|--------------------|---|
| a | = semimajor axis of object |
| E(a) | = expected chord length of a sphere of radius a |
| $E(t_c)$ | = characteristic encounter time |
| $F(r_c)$ | = Weibull probability at collision radius |
| F(x) | = Weibull cumulative probability distribution |
| | function |
| f(x) | = Weibull probability density function |
| h_{as} | = orbit apogee of Space Station |
| h_{ps} | = orbit perigee of Space Station |
| I, i | = orbital inclination |
| J_2 | = Earth oblateness coefficient |
| L | = latitude |
| L_1 | = lower latitude bound |
| L_2 | = upper latitude bound |
| N | = encounter rate |
| P_{col} | = probability of collision |
| $P(L_1, L_2)$ | = probability of being between the latitude bands L_1 |
| | and L_2 |
| $P(r_p, r)$ | = probability of being between a given radius and |
| | the periapsis |
| $P(r_1, r_2)$ | = probability of being between two given radii |
| P(0, L) | = probability of being within a latitude band from |
| | equator to latitude, L |
| r | = radius |
| r_a | = apoapsis radius |
| r_c | = collision radius |
| r_p | = periapsis radius |
| r_1 | = lower radius bound |
| r_2 | = upper radius bound |
| T_c | = Vedder formulation of expected time between |
| | collisions |
| T_m | = Weibull formulation of expected time between |
| | collisions |
| V_r | = relative speed |
| $V_{ m SS}$ | = speed of Space Station |
| β | = Weibull scale parameter |

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 Γ = statistical gamma function ΔT_{tier} = duration of tier passage

 $\rho_{\rm sd}$ = spatial density of orbital particulates

 τ = Weibull shape parameter

Introduction

LL spacecraft face the threat of collision with other objects in space; however, when large-population constellations are used, the threat becomes more complex. Satellites in large constellations must endure more than the threat due to background space debris and meteoroids. They must endure the threat of collisions between other constellation members, the threat of collisions with the concentrated debris of destroyed constellation members, and the threat of collisions due to objects whose orbits are very similar to the constellation's and that are repeatedly raised and lowered through the constellation altitude. The purpose of the study was to provide an analysis of the threats to a sample large-population constellation projected to the year 2002 from resident space objects and a Space Station (SS) sized object passing through the constellation.

This study first evaluates the space environment to which the sample constellation is exposed. The environment of particles ≥ 10 cm is determined from the U.S. Space Command (USSPACECOM) Catalog. The environment for particles < 10 cm is determined using the NASA EVOLVE computer program. Next, the SS/Sample Constellation collision assessments are performed using the particle-ina-box (PIB) method and the Weibull probability distribution (based on propagation simulations and Monte Carlo simulations). Finally, the expected time between encounters for given keep-out distances is computed for the SS/Sample Constellation. This study consists of two parts: 1) space environment and 2) encounter simulations.

Reference Constellation Environment Analysis Current Space Environment

To better understand the environment that a satellite constellation creates, an environmental determination was performed. It was based on the current background environment within a sample constellation's potential orbital range and the projected environment for the given constellation. The background environment consists of spacecraft, rocket bodies, and debris. The sample constellation consisted of two tiers of satellites: tier 1 containing 930 satellites in 440-km circular orbits at 50-deg inclination, and tier 2 containing 270 satellites in 450-km circular orbits at 80-deg inclination. The satellites were allowed to vary about the normal orbit by ± 1 km. These orbits were representative of the Brilliant Pebbles constellation, which was in the altitude range of the Space Station Freedom. The study approach, however, is applicable to any other constellation and the International Space Station, for example, should they

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occupy the same altitude range in low Earth orbit. The environmental evaluation included the following: the determinations of the number of objects vs eccentricity, the number of objects vs inclination, and the spatial density vs altitude and latitude. Each evaluation determined the environment for each category of background objects: the summation of the background objects, the sample constellation, and the combination of the total background and sample constellation. Background object orbital parameters were taken from the USSPACECOM Catalog.

The region of interest for an environmental evaluation was determined by the location of the sample constellation. That region was occupied by the satellites of the sample constellation, by the constellation's launch vehicle and associated debris, and by other spacecraft, rocket bodies, and debris. The current trackable population with objects that were ≥10 cm in diameter was separated into three categories: space debris, rocket bodies, and spacecraft. This information was obtained from the USSPACECOM Catalog. The catalog was scanned for all objects that were within the region of interest. The objects were then placed into a data file to be used by the environment determination programs. The scan revealed the type of objects that existed within the region of interest in the categories of debris, rocket bodies, and spacecraft. Input files were produced for the objects within the range of interest for each category.

Spatial Density vs Latitude and Altitude

The spatial density vs latitude and altitude was determined using Dennis' approach, ¹ for each object as the product of the probability that an object is within a given latitude range, 1 deg, and of the probability that an object is within a given altitude range, 1 km, divided by the volume element for that region. The geometry of that volume element (as shown in Fig. 1) is a ring that is 1 deg wide, 1 km thick and that lies above the examined latitude at the

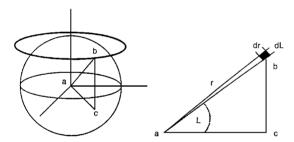


Fig. 1 View of the volume cell for a given latitude and radius.

examined altitude. The volume was computed as volume = (ring length) (width) (thickness),

$$V = 2\pi \left[\left(\frac{2r + 1 \text{ km}}{2} \right) \cos \left(\frac{2L + 1 \text{ deg}}{2} \right) \right]$$

$$\times \left[\left(\frac{2r + 1 \text{ km}}{2} \right) (1 \text{ deg}) \left(\frac{\pi}{180 \text{ deg}} \right) \right] [1 \text{ km}]$$
 (1)

The values of r and L are taken as twice their values plus 1 (km or deg), quantity divided by 2, so that the cross section of the length is located at the midpoints of the latitude and radius ranges. The total spatial densities are the sums of the densities for each object at the appropriate locations. The probability that an object is between a given altitude and the perigee of the object is given by

$$P(r_p, r) = \frac{1}{2} + \frac{1}{\pi} \sin^{-1} \left[\frac{2(r-a)}{(r_a - r_p)} \right] - \frac{1}{a\pi} \sqrt{(r_a - r)(r - r_p)}$$
(2)

The probability that an object is between a given altitude, r_1 , and the next step altitude value, $r_2=r_1+1\,$ km, is given by

$$P(r_1, r_2) = \frac{1}{a\pi} \left\{ \sqrt{(r_a - r_1)(r_1 - r_p)} - \sqrt{(r_a - r_2)(r_2 - r_p)} \right\}$$

$$+ \frac{1}{\pi} \left\{ \sin^{-1} \left[\frac{2(r_2 - a)}{(r_a - r_p)} \right] - \sin^{-1} \left[\frac{2(r_1 - a)}{(r_a - r_p)} \right] \right\}$$
(3)

The probability that an object is between the equator and a given latitude is given by

$$P(0,L) = \frac{1}{\pi} \sin^{-1} \left[\frac{\sin(L)}{\sin(i)} \right]$$
 (4)

The probability of being between a given latitude, L_1 , and the next step value, $L_2 = L_1 + 1$ deg, is given by

$$P(L_1, L_2) = \frac{1}{\pi} \left[\sin^{-1} \left(\frac{\sin(L_2)}{\sin(i)} \right) - \sin^{-1} \left(\frac{\sin(L_1)}{\sin(i)} \right) \right]$$
(5)

DENSITY, a Fortran program, was created to compute the spatial density of objects in Earth orbit, in objects per cubic kilometer, as a function of orbital radius and of latitude. The program evaluated the spatial density at a given latitude and radius for all objects that passed through the desired radius range. The results were easier to visualize when they were graphed three dimensionally (see Fig. 2 for the graph of background objects).

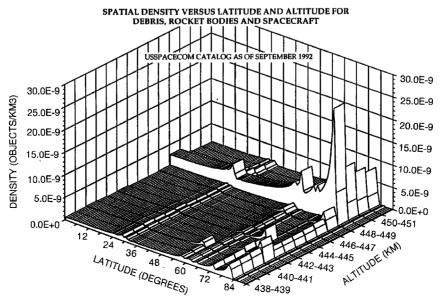


Fig. 2 Three-dimensional view of spatial density calculation for background.

Projected Space Environment

The projected population was separated into two categories: 1) the fully operational sample constellation alone and 2) the fully operational sample constellation with the expected resident space object background population at any given time during its operational phase. The projected population is based on the present population with an assumed 5% per year growth rate (as has been seen in historical launch activity). Taking the 5% per year growth rate over the number of years from September 1992 yielded a factor of increase over the present population. The background population spatial density values for the September 1992 spatial density data were to be multiplied by that factor to determine the expected values in the year of interest.

Projected population estimates performed were based on two sources: the trackable population (≥10 cm objects) as taken from USSPACECOM radar measurements and the untrackable population (<10 cm objects) as determined by Massachusetts Institute of Technology optical observations and by NASA on-orbit vehicle impact assessments. The Aerospace Corporation's projected spatial density determinations used the USSPACECOM Catalog of trackable objects along with an assumed space debris growth rate. NASA used the USSPACECOM Catalog along with the untrackable population data to perform projected space debris flux and impact determinations according to their space debris model. NASA's model for meteoroid flux was compared with the space debris flux data. Finally, the expected launch activities for the sample constellation and for the planned communication satellite constellations were evaluated.

The NASA space debris flux projection model² that was used to determine the flux environment accounted for the newest assumed growth rates as provided by Kessler³: 2% compound from 1990 for smaller objects and 5% linear from 1990 for larger objects. The meteoroid flux model⁴ that was created in 1969 by Cour-Palais provided a temporally static view of the micrometeoroid environment. Both environmental models were used to determine the space object flux to which a constellation would be subject, as seen in Fig. 3.

The spatial density at the sample constellation altitude range would be affected by launch vehicles and their debris. DENSITY runs showed that the most significant contributors to this density increase will be launch vehicles for the sample constellation itself and for several planned communication constellations. For the sample constellation, the density was equivalent to the maximum background density; however, it was concentrated at the same regions that the sample constellation inhabited. The communication constellations' launch vehicle densities were used as guideline values because their launch configurations were estimated. The launch activity for the communication satellite constellations did not significantly increase the threat to the sample constellation.

There are two problems with this analysis: 1) There will be launch vehicle debris and 2) the sizes of the final stages will be very large. The number of debris pieces released per launch vehicle would have a multiplicative effect on the given total density. The problem of stage size is not reflected in the spatial density calculation. The

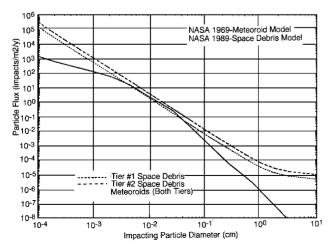


Fig. 3 Flux on sample spacecraft vs impacting particle size, year 2002.

calculation provides the number of objects per cubic kilometer. This means that in a given volume of space of $1\,\mathrm{km^3}$, a small particle would be counted the same as one object with a volume of $1\,\mathrm{km^3}$. For a vehicle passing through a given volume of space that is much greater than the volume of the vehicle, there is a much greater chance (100%) of hitting a large object that fully consumes that volume of space than there is of hitting one small object that only consumes a minuscule fraction of that volume. Because the spatial density values do not relate the full consequences to objects in orbit, mission planners should not be content with seemingly acceptable density values. The sample constellation launch vehicles should be removed from orbit as soon as possible. Satellite insertion and removal should be as debris free as possible to reduce the threat to the sample constellation.

Constellation/SS Collision Probability Analysis

SS/Sample Constellation Collision Assessment

This section examines the probability of collision between the SS and a satellite of the sample constellation per passage of the SS through the sample constellation. The SS operating altitude is 466-425 km with reboost from lower altitude occurring every 90 days. The inclination of the SS orbit is 28.5 deg. The SS orbit apogee and perigee altitudes straddle both tiers of the sample constellation. For an assumed SS altitude decay rate of 0.456 km/day, the effective SS dwell time within each 2-km-thick tier is approximately 2.5 days. The dwell time represents the fraction of time that the SS dwells within the altitude band of each sample constellation tier, as shown in Fig. 4 for tier 2. The top part depicts the two altitude tiers of the sample constellation with the SS altitude range currently overlapping tier 2 and moving lower at a rate of 0.456 km/day. The lower part depicts the SS dwell time fraction vs the number of days it takes to traverse tier 2. The results for tier 1 are essentially the same. For tier 1, the dwell time is zero at $h_{ps} = 441 \,\mathrm{km}$, maximum at $h_{ps} = 436.6$ km, and zero again at $h_{as} = 439$ km altitude when the SS exits the tier 1 altitude band. For tier 2, the dwell time is zero at $h_{ps}=451\,\mathrm{km}$, maximum at $h_{ps}=446.6\,\mathrm{km}$, and zero again at $h_{as}=449\,\mathrm{km}$ altitude when the SS exits the tier 2 altitude band.

The spatial density that the sample constellation creates will be two to three orders of magnitude greater than that of the background resident space objects. One of the most serious problems with a density increase in this region of space is that the SS was to be passing through it, thus posing great danger to the station, its occupants, and subsequently all other satellites of the sample constellation. To determine the potential of this threat, an evaluation of the probability of collision per altitude range passage was performed. The evaluation was based on the PIB approach, where the probability of collision is a function of the spatial density of the region being traversed, the speed at which traversal occurs, cross-sectional area of object traversing the region, and the time of traversal. The equation and values used were⁵

$$P_{\rm col} = \rho_{\rm sd} \cdot V_{\rm SS} \cdot A_{\rm SS} \cdot \Delta T_{\rm tier} \tag{6}$$

The value for spatial density was determined as discussed earlier. To determine the average density encountered, the spatial density vs latitude output was integrated to locate the latitude, which divided

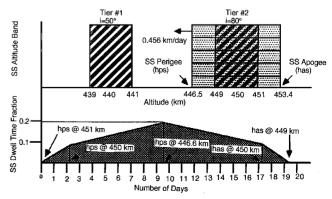


Fig. 4 SS dwell time fraction of tier 2 passage.

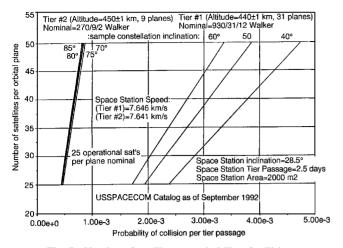


Fig. 5 Number of satellites vs probability of collision.

the integrated value in half. This value for each altitude occurred at 15 deg; at 440 km, the density was 0.706719E-06 objects/km³; at 450 km, the density was 0.156051E-06 objects/km³. Although the objects that created the spatial density were in dynamic orbits, a temporally static view of the environment was used and, thus, the spatial density was in the form of a static field. Because the density field was assumed to be static, SS speed through the field is its orbital speed: 7.646 km/s at 440 km and 7.641 km/s at 450 km. SS's cross-sectionalarea was given as $2000\,\mathrm{m}^2$, or $0.002\,\mathrm{km}^2$. As already shown, SS's effective time of passage through each of the tiers was computed to be 2.5 days. These values provided the following results: tier 1 $P_\mathrm{col}=0.002334$ and tier 2 $P_\mathrm{col}=0.000515$.

An extension of the work that was performed to determine the sample constellationthreat to the SS was the variation of the inclination of and the population of the sample constellation. The number of satellites per plane was varied from 25 to 50 in five-satellite increments. The results appear in Fig. 5.

Weibull Probability Distribution

Several studies published in the technical literature use statistical approaches to determine the probability that two objects will collide in orbit. One such study⁶ uses concepts from the asymptotic theory of extreme-order statistics to the distribution of distances from a target satellite to the nearest orbiting object at random points in time. The distribution of close approach distances is obtained by a Monte Carlo sampling technique for all objects, which are assumed to be moving in specified Keplerian orbits. A considerable degree of realism is apparent in the approach, which retains the kinematical information peculiar to the object population of interest.

In the present analysis, the approach described by Vedder and Tabor⁶ is utilized but modified somewhat to include perturbative effects due to Earth oblateness in the Monte Carlo sampling procedure. Also simplified is the expression for the collision rate, which provides better insight and utility of the results obtained. The analytical/numerical procedure is then applied to compute the probability that the SS will collide with a member of a reference constellation. The results are compared with those obtained from a miss-distance determination software ANCAS⁷ and are found to be in excellent agreement.

The probability of collision between a spacecraft and a member of a population of objects can be computed using a Weibull probability distribution fit to the encounter statistics generated by the Monte Carlo method. The basic concept of this approach is to record the frequency of the ranges between the spacecraft and the nearest object at a large number of different times and, thus, obtain the distribution of these ranges, which represents the relative fraction of time that the object nearest to the spacecraft is a given distance away. This distribution tapers off at small values of range and vanishes at zero range. Sufficiently small values of range represent situations in which the nearest object is close enough to the spacecraft to collide with it. Thus, the estimation of the Weibull distribution function that best fits the observable ranges is equivalent to

estimating the probability of collision in the virtually unobservable collision proximity distances.

The generalized extreme value distribution method approach used consisted of the following steps. 1) Generate (via a Monte Carlo or orbit propagation) range distribution between the spacecraft and the nearest object. 2) Fit a Weibull probability density function to the empirical data set. 3) Compute a characteristic encounter time and an estimated time to collision between the spacecraft and a constellation member.

Step 1 requires modeling orbits of the satellites of interest. Steps 2 and 3 require selection of the shape and scale parameters τ and β , respectively, for the Weibull probability density function of the form

$$f(x) = (\tau/\beta)(x/\beta)^{\tau-1} \exp\{-(x/\beta)^{\tau}\}$$
 (7)

with the resulting Weibull cumulative probability distribution function, as generated from an integration of the probability density function with the constant of integration being set to a value of 1,

$$F(x) = \int f(x) dx = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{\tau}\right] \approx \left(\frac{x}{\beta}\right)^{\tau}$$
 for $\left(\frac{x}{\beta}\right) \ll 1$ (8)

The encounter rate can be defined as

$$N = \frac{F(r_c)}{E(t_c)} \tag{9}$$

where

$$E(t_c) = E(a)/V_r \tag{10}$$

and where E(a) is numerically equal to one-quarter of the circumference of a circle. This formulation results in an expression for the expected time to collision

$$T_m = 1/N = (\pi r_c/2V_r)(\beta/r_c)^{\tau}$$
 (11)

For comparison, the corresponding value of the time to collision⁶ is

$$T_c = \left(1 + \frac{1}{\tau}\right)^{-1} \frac{\beta \sqrt{\pi}}{V_r} \left\{ \frac{\Gamma(\tau/2)}{\Gamma[(\tau+1)/2]} \right\} \left(\frac{r_c}{\beta}\right)^{1-\tau} \tag{12}$$

Numerical results obtained from Eq. (11) are generally in very good agreement with those obtained from Eq. (12). Equation (11), however, is simpler in form than Eq. (12) and, thus, provides improved insight into the physics of close encounters.

Propagation Simulation

The SS encounter statistics with the constellation satellites resulting from the SS altitude decay through the constellation were generated by propagation. The propagation method determined relative range at 1-min intervals between SS and the nearest spacecraft for 1 day (24 h). The effects of Earth oblateness were included in the orbital propagation model developed. The distribution of the relative range between SS and the closest spacecraft at 1-min intervals for 1 day (1440 points) is presented in Fig. 6.

Monte Carlo Simulation

The propagation of each of the 930 satellites and the SS was performed on a personal computer. A typical simulation run, however, required several days. To reduce the run time, an alternative simulation based on random sampling of the relative range between the SS and the nearest satellite was performed. The relative range and corresponding relative velocity were obtained for each of the 8000 trials taken. For each of the 8000 trials, the mean anomaly and right ascension of the ascending node were randomly selected for the SS and each of two constellation tiers independently. The relative spacing of the satellites in mean anomaly and node remained constant. The relative range distribution from the Monte Carlo simulation is shown in Fig. 7. The good agreement between the propagation and Monte Carlo distributions is apparent in Figs. 6 and 7.

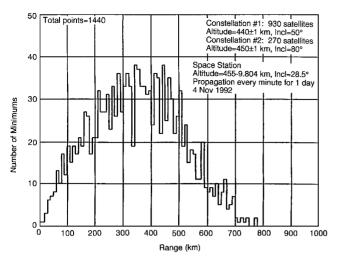


Fig. 6 Ranges between satellites and SS.

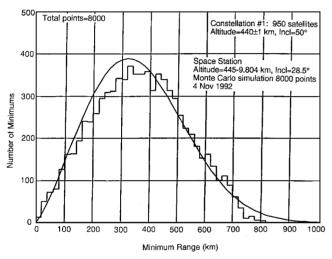


Fig. 7 Weibull probability density function: $\beta = 412.5$ km, $\tau = 2.30$.

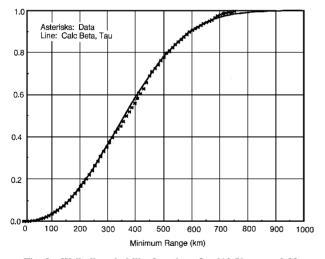


Fig. 8 Weibull probability function: β = 412.5 km, τ = 2.38.

Weibull Analysis Example

Consider the range distribution function of Fig. 7. The distribution data approximated by Eq. (7) with $\beta=412.5$ km and $\tau=2.38$ is overlaid. The corresponding cumulative probability function for the data and the approximation are shown in Fig. 8.

The expected mean time to collision from Eq. (11) is

$$T_m = \frac{\pi (0.025)}{2 \times 7.5} \left(\frac{412.5}{0.025} \right)^{2.38} = 661 \,\text{days}$$
 (13)

Table 1 Minimum range between SS and satellites in tier 1 within 10-km radius of SS for 1 day

| Encounter number | Encounter range, km | Encounter time, h:min:s |
|---------------------|------------------------|-------------------------|
| 1 | 5.61 | 3:39:50 |
| 2 | 7.39 | 5:25:5 |
| 3 | 3.70 | 8:6:15 |
| 4 | 8.08 | 9:51:40 |
| 5 | 4.78 | 14:15:5 |
| 6 | 9.73 | 19:53:30 |

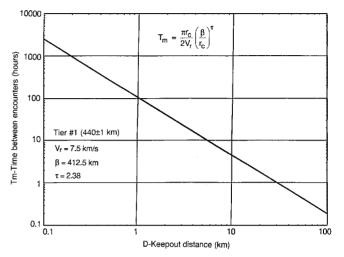


Fig. 9 SS/satellite encounter rate.

where the SS projected frontal radius $r=25\,\mathrm{m}$, relative velocity $V_r=7.5\,\mathrm{km/s}$, and the Weibull fit parameters of Fig. 7 are used. The probability of collision per pass through the constellation is, therefore, $2.5/661\approx0.4\%$. This compares with $P_{\rm col}\approx0.2\%$ obtained previously using the PIB approach.

ANCAS Simulation

The plot of Eq. (11) for different values of r corresponding to arbitrary keep-out distances D is shown in Fig. 9 for tier 1 constellation. It is apparent that the expected (mean) time between encounters for D=1 km is 100 h and 4.07 h within a sphere of 10-km radius.

A good agreement with these results has been obtained using close approach software ANCAS. The ANCAS simulation results are listed in Table 1. These results provide the minimum range and the time of entry and exit into a sphere of 10-km radius centered on the SS for a period of 1 day using a step size of 5 s. A total of six encounters was observed compared to the 5.9 encounters expected by Eq. (11).

Summary and Conclusions

The space environment was determined for the region of space that the sample constellation will occupy. The population, spatial density, and flux were determined for all latitudes for the radial range of 6817–6829 km (altitude = 439 – 451 km). The population was determined for all inclinations and for all periodic eccentricities. The population determination and spatial density were based on the USSPACECOM Catalog as of September 1992, the presence of the sample constellation, the launch activity for the sample constellation, and the launch activity for planned communications satellite constellations. The flux determination was based on NASA's space debris and meteoroid models. The spatial density determination indicated that the presence of the sample constellation increased the density by a factor of 100–1000 over the background density, assuming that there was no collision avoidance control and that the characteristic altitude of each orbit was the same.

One of the most serious problems with a density increase in this region of space is that the SS would be passing through it, thus posing a hazard to the station, its occupants, and subsequently all other

sample constellation satellites. The evaluation of the collision probability, based on the PIB approach, was found to be on the order of 0.2%. The sample constellation's threat to the Space Station was also determined with a variation of the sample constellation's population.

A modification of a method based on Monte Carlo sampling of close approach distances between satellites has been used to determine the probability that the SS would collide with a member of a multisatellite constellation in a drift through the nominal altitude of the constellation. Also, the encounter rate within a 10-km radius of the SS was determined, which agreed with the results obtained from a miss-distance determination software package ANCAS. The probability of SS collision with a satellite per pass through the tier 1 constellation was found to be 0.4% based on the Monte Carlo analysis.

Implications for large-population satellite constellations are that the constellation should not be deployed in regions of space frequented by manned vehicles and that recovery or deorbit of launch components and nonfunctioning spacecraftis prudent. It should also be recognized that particle sizes from 0.1 to 1.0 cm will be particularly important in the future, due to spacecraft with larger structures and longer on-orbit times. The sample constellation will satisfy both of these requirements: 1) There will be many of them (equivalently, larger structure) and 2) they will spend approximately 10 years on orbit (longer times).

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